Hydraulics

<u>3rd Year civil</u>

<u>First Term (2009 - 2010)</u>

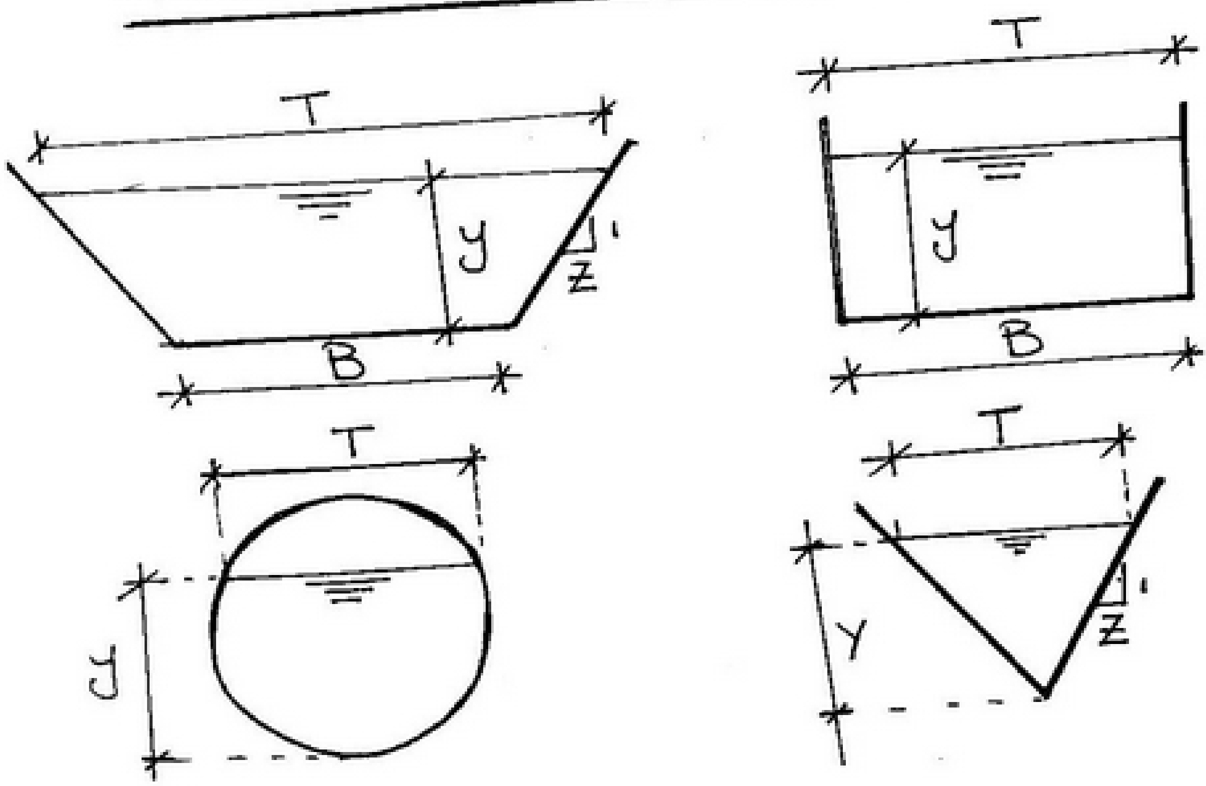
Chapter (   )

2009 - 2010

بسم الله الرحمن الرحيم

Ch(3): Geometric properties of open channel

- Geometric elements of channel section:



B : bottom width of section.

y : water depth in section.

T: top width of section

y_h : mean hydraulic depth $= \frac{A}{T}$

A : area of section.

P : premeter of section.

R : Hydraulic radius $= \frac{A}{P}$

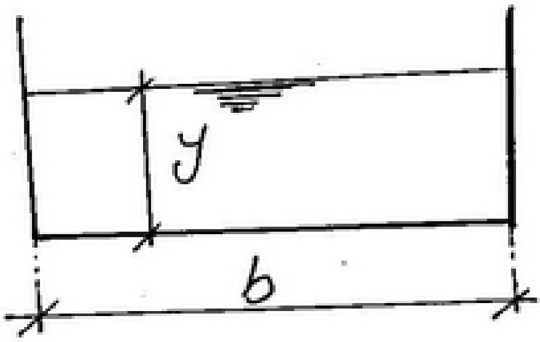
Z : section factor $= A \times \sqrt{y_h}$

Very wide section :

يسمى القطاع عريض جداً

إذا كان

$$\boxed{b \geqslant 10\,y}$$



$$A = b \times y \quad , \quad P = b + 2y$$

$$R = \frac{A}{P} = \frac{b \cdot y}{b + 2y}$$

$$R = \frac{b}{\frac{b}{y} + 2} \qquad y \to \infty$$

$$\boxed{R = \frac{b}{2}}$$

Best Hydraulic section (B.H.S)

القطاع الهيدروليكى الأمثل

يعرف على انه القطاع الذى يعطى أقصى تصرف مع أقل محيط مبتل عند ثبات مساحة القطاع وميله (S) ومعامل الخشونة داخله.

بعض المرادفات لكلمة B.H.S

- most economical section.
- section with minimum Lining.
- section of max discharge.
- section of min. excavation.
- section with min ~~slope~~ slope.

for (b) is big value divide by (b)

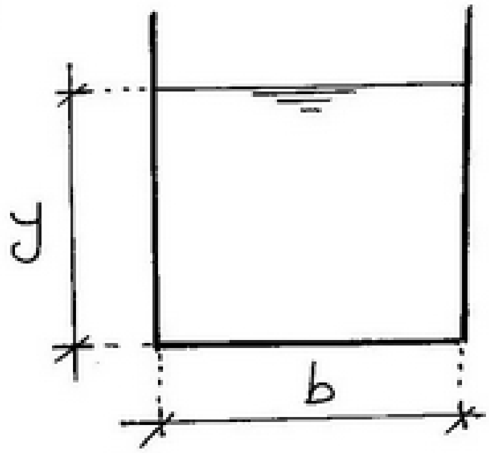
$$\therefore \quad R = \frac{y}{1 + \frac{2y}{b}} \qquad b \longrightarrow \infty$$

$$\therefore \quad \boxed{R = y}$$

Very deep section:

يسمى القطاع عميق جداً
إذا كان

$$\boxed{y \geqslant 10\, b}$$



$$A = b \cdot y \quad , \quad P = b + 2y$$

$$R = \frac{A}{P} = \frac{b \cdot y}{b + 2y}$$

Divide by (y)

Steps to prove B.H.S:

خطوات عمل القطاع الهيدروليكي الأمثل

١- كتابه معادلة المساحة والمحيط للقطاع.

٢- من معادلة المساحة نحصل على متغير بدلالة الآخر.

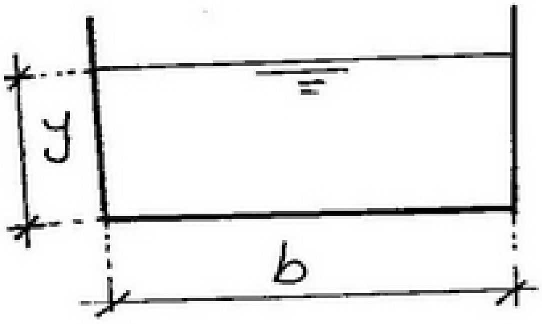
٣- نعوض عن المتغيرات في معادلة المحيط

٤- نفاضل معادلة المحيط ونساويها بالصفر $\frac{dp}{d?} = 0$

٥- نوجد العلاقة النهائية.

Rectangular section:

- $A = b \times y$ ①
- $P = b + 2y$ ②



from ①

$$b = \frac{A}{y}$$

subis. in ②

$$P = \frac{A}{y} + 2y$$

for B.H.S $\frac{dp}{dy} = 0$

$$0 = -\frac{A}{y^2} + 2$$

$$\frac{A}{y^2} = 2 \Rightarrow A = 2y^2$$

$$b.\cancel{y} = 2y^{\cancel{2}}$$

$$\boxed{b = 2y}$$

for $R = \frac{A}{P} = \frac{b.y}{b+2y}$

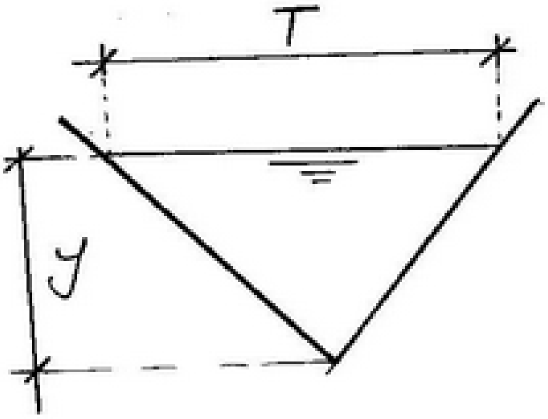
$$R = \frac{2y \times y}{2y+2y} = \frac{2y^2}{4y}$$

$$\boxed{R = \frac{y}{2}}$$

Triangular section:

- $A = \frac{1}{2} \cdot T \cdot y$ ①

- $P = 2\sqrt{\left(\frac{T}{2}\right)^2 + y^2}$ ②



from ①

$$T = \frac{2A}{y}$$

subis. in ②

$$P = 2\sqrt{\left(\frac{2A}{2y}\right)^2 + y^2} = 2\sqrt{\frac{A^2}{y^2} + y^2}$$

$$= 2\sqrt{\frac{A^2 + y^4}{y^2}}$$

$$P = \frac{2}{y}\sqrt{A^2 + y^4}$$

for B.H.S $\frac{dP}{dy} = 0$

$$0 = \frac{\cancel{2}}{y} \times \frac{4y^3}{\cancel{2}\sqrt{A^2+y^4}} - \sqrt{A^2+y^4} \times \frac{2}{y^2}$$

$$\frac{4y^2}{\sqrt{A^2+y^4}} = \frac{2\sqrt{A^2+y^4}}{y^2}$$

$$\therefore 4y^4 = 2(A^2+y^4)$$

$$4y^4 = 2A^2 + 2y^4$$

$$\cancel{2}y^4 = \cancel{2}A^2$$

$$y^4 = A^2$$ بأخذ جذر الطرفين

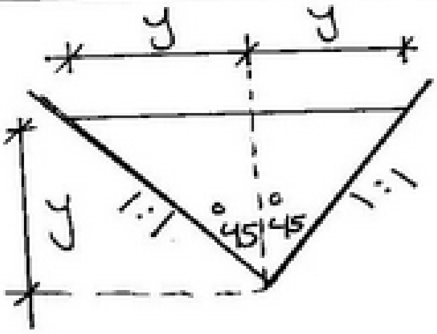
$$\therefore \boxed{y^2 = A}$$

subis in ①

$$\frac{T \cdot \cancel{y}}{2} = y^{\cancel{2}} \Rightarrow$$

$$\therefore \boxed{T = 2y}$$

يكون القطاع المثلث B.H.S
إذا كانت زاوية رأس
المثلث $(90°)$ أو أن
الميل الجانبي له $(1:1)$

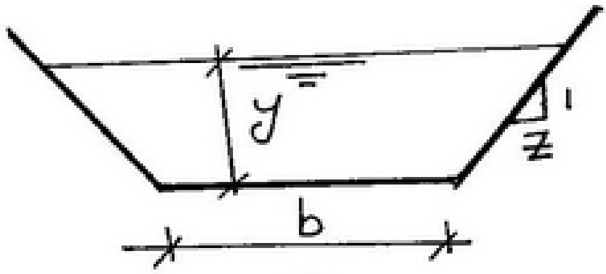


$$\because \quad R = \frac{A}{P} = \frac{2y^2}{2} \times \frac{y}{2\sqrt{2y^4}}$$

$$R = \frac{y^3}{2\sqrt{2y^4}}$$

$$\boxed{R = \frac{y}{2\sqrt{2}}}$$

Trapizoidal section



- $A = (b + Zy)\,y \longrightarrow (1)$
- $P = b + 2y\sqrt{1+Z^2} \longrightarrow (2)$

from (1)

$$b = \frac{A}{y} - Zy$$

subis. in (2)

$$\therefore P = \frac{A}{y} - Zy + 2y\sqrt{1+Z^2}$$

for Z = Const. :

for B.H.S $\frac{dP}{dy} = 0$

$$0 = \frac{-A}{y^2} - Z + 2\sqrt{1+Z^2}$$

$$\therefore \quad 2\sqrt{1+Z^2} = \frac{A}{y^2} + Z \longrightarrow ③$$

from ① in ③

$$2\sqrt{1+Z^2} = \frac{(b+Zy)y}{y^2} + Z$$

$$2\sqrt{1+Z^2} = \frac{(b+Zy)y + Zy^2}{y^2}$$

$$2y^2\sqrt{1+Z^2} = b.y + 2Zy^2$$

Divide by y

$$\boxed{2y\sqrt{1+Z^2} = b + 2Zy} \longrightarrow ④$$

for $R = \frac{A}{P} = \frac{(b+Zy)y}{b+2y\sqrt{1+Z^2}}$

$$R = \frac{(b+Zy)y}{b+b+2Zy} = \frac{\cancel{(b+Zy)}y}{2\cancel{(b+Zy)}}$$

$$\boxed{R = \frac{y}{2}}$$

For y = Const and Z Variable :-

$$P = \frac{A}{y} - Zy + 2y\sqrt{1+Z^2}$$

for B.H.S $\frac{dP}{dZ} = 0$ for Const y

$$0 = 0 - y + 2y \times \frac{2Z}{2\sqrt{1+Z^2}}$$

$$\cancel{y} = \frac{2Z\cancel{y}}{\sqrt{1+Z^2}}$$

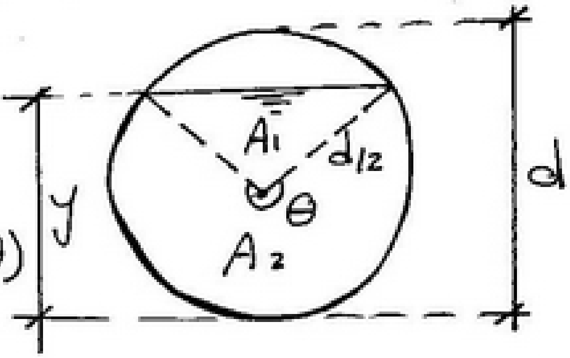
$$2Z = \sqrt{1+Z^2}$$ بتربيع الطرفين

$$4Z^2 = 1 + Z^2$$

$$3Z^2 = 1$$

$$Z^2 = \frac{1}{3} \Rightarrow \boxed{Z = 1/\sqrt{3}}$$

Circular section :



$$- \because \quad A = A_1 + A_2$$

$$\therefore \quad A_1 = \frac{1}{2}\left(\frac{d}{2}\right)^2 \times \sin(360-\theta)$$

$$A_1 = -\frac{d^2}{8}\sin\theta$$

$$A \begin{cases} \frac{\pi}{4}d^2 \longrightarrow 360^\circ \\ A^2 \longrightarrow \theta \end{cases}$$

$$A_2 = \frac{\pi d^2}{4} \times \frac{\theta}{360}$$

$$= \frac{\pi d^2}{4} \times \frac{\theta}{2 \times 180}$$

$$P \begin{cases} \pi d \longrightarrow 360^\circ \\ P \longrightarrow \theta \end{cases}$$

$$\because \quad \frac{\pi\theta}{180} = \theta_r$$

$$\therefore \quad A_2 = \frac{d^2}{8}\theta_r$$

$$\therefore \quad A = \frac{d^2}{8}\theta_r - \frac{d^2}{8}\sin\theta$$

$$\therefore \quad A = \frac{d^2}{8}(\theta_r - \sin\theta) \longrightarrow (1)$$

$$\because \quad P = \pi \cdot d \times \frac{\theta}{360}$$

$$\therefore \quad P = \frac{\pi \theta \cdot d}{180 \times 2}$$

$$P = \frac{d}{2} \cdot \theta_r \longrightarrow (2)$$

from (1)

$$d = \sqrt{\frac{8A}{\theta_r - \sin\theta}}$$

subis in (2)

$$P = \frac{\theta_r}{2} \sqrt{\frac{8A}{\theta_r - \sin\theta}}$$

$$\therefore \quad P = \frac{1}{2} \sqrt{\frac{8A\,\theta_r^2}{\theta_r - \sin\theta}}$$

for B. H.S $\quad \frac{dP}{d\theta} = 0$

$$0 = \frac{1}{2} \times \frac{1}{2\sqrt{\frac{8A\theta_r^2}{\theta_r - \sin\theta}}} \times \frac{8A\theta_r^2 \times (1 - \cos\theta) - (\theta_r - \sin\theta) \times 16 A \theta_r}{(\theta_r - \sin\theta)^2}$$

$$\therefore \quad \cancel{8}\cancel{A}\theta_r^{\cancel{2}} (1 - \cos\theta) = \overset{2}{\cancel{16}}\cancel{A}\cancel{\theta_r} (\theta_r - \sin\theta)$$

$$\therefore \boxed{\theta_r(1-\cos\theta) = 2(\theta_r - \sin\theta)}$$

by trial. $\theta = \pi = 180°$

for $R = \frac{A}{P}$

$$\therefore \boxed{R = \frac{d}{4}}$$

section	Condition for B.H.S
Rectangular	$b = 2y$, $R = y/2$
Triangular	$Z = 1$, $R = y/2\sqrt{2}$
Trapizoidal	Z const $R = y/2$ y const . $Z = 1/\sqrt{3}$
Circular	$\theta_r = \pi$, $\theta = 180$ $R = d/4$

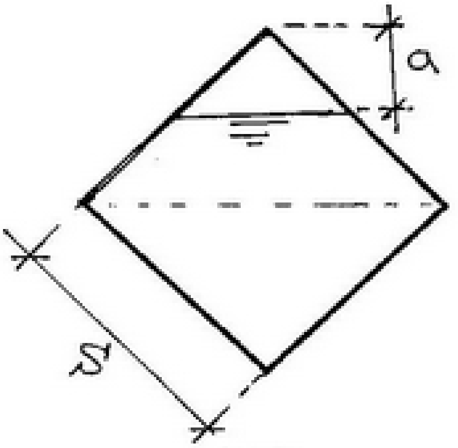
Note for any Circular section to get

V_{max} $$\boxed{P \cdot \frac{dA}{d\theta} = A \times \frac{dP}{d\theta}}$$

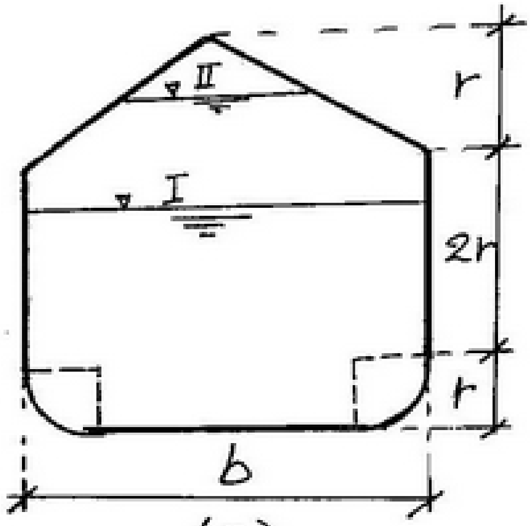
Q_{max} $$\boxed{2.5\,P \cdot \frac{dA}{d\theta} = A \cdot \frac{dP}{d\theta}}$$ (Manning)

$$\boxed{3\,P \cdot \frac{dA}{d\theta} = A \cdot \frac{dP}{d\theta}}$$ (chezy)

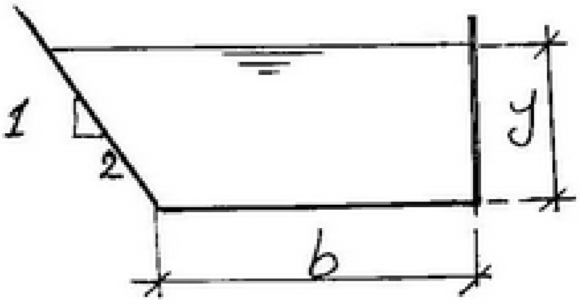
For the following section find the Conditions for B.H.S



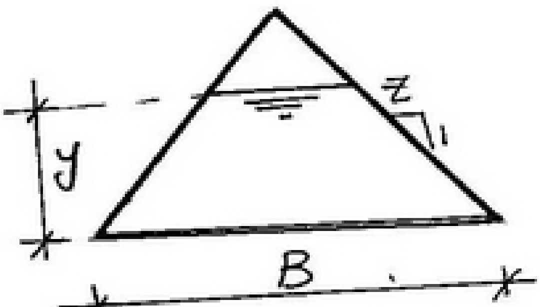
(b)



(a)



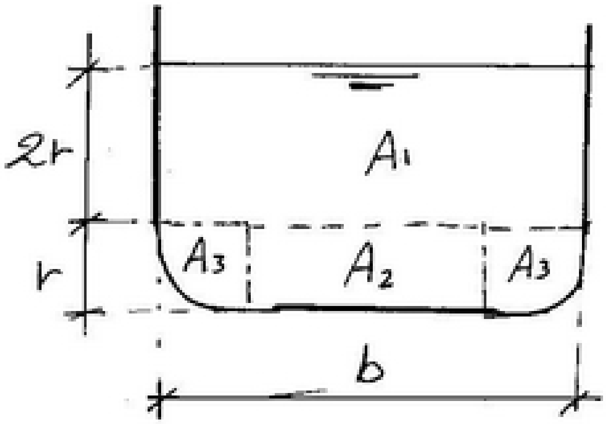
(d)



(c)

(a) : Stage I :

$$A = A_1 + A_2 + 2 A_r$$



$$- A_1 = 2br$$

$$- A_2 = (b - 2r) r$$

$$= br - 2r^2$$

$$- A_3 = \frac{\pi r^2}{4} \times 2 = 1.57 r^2$$

$$A_t = 2br + br - 2r^2 + 1.57 r^2$$

$$\therefore A_t = 3br - 0.43 r^2 \longrightarrow (1)$$

$$P = 4r + b - 2r + \frac{2\pi r}{2}$$

$$\therefore P = 5.14 r + b \longrightarrow (2)$$

from (1)

$$b = \frac{A + 0.43 r^2}{3r} = \frac{A}{3r} + 0.14 r$$

subis. in ②

$$\therefore P = 5.28\,r + \frac{A}{3r}$$

for B.H.S $\frac{dP}{dr} = 0$

$$0 = 5.28 - \frac{A}{3r^2}$$

$$\therefore \frac{A}{3r^2} = 5.28$$

$$\therefore A = 15.84\,r^2$$

$$\therefore 3br - 0.43\,r^2 = 15.84\,r^2$$

$$3br = 16.27\,r^2$$

$$\therefore \boxed{b = 5.42\,r} \quad \#$$

$$\therefore \text{for } R = \frac{A}{P} = \frac{3br - 0.43r^2}{5.14r + b}$$

$$\therefore R = \frac{3 \times 5.42r^2 - 0.43\,r^2}{5.14\,r + 5.42\,r^2} = \frac{15.83\,r^2}{10.56\,r}$$

$$\boxed{R = 1.50\,r} \quad \#$$